

# Comment on “Prospects for a new search for the electron electric-dipole moment in solid gadolinium-iron-garnet ceramics”

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In a recent paper [A. O. Sushkov, S. Eckel and S. K. Lamoreaux, Phys. Rev. A **79**, 022118 (2009)] the authors measured the EDM-induced magnetization  $M$  that is given by Eq. (1) in their paper. Such an expression for  $M$  is a consequence of the generally accepted opinion that both dipole moments, a MDM  $\mathbf{m}$  and an EDM  $\mathbf{d}$ , are proportional to the spin  $\mathbf{S}$ . Recently [T. Ivezić, Phys. Scr. **81**, 025001 (2010)] the Uhlenbeck-Goudsmit hypothesis is generalized in a Lorentz covariant manner using the four-dimensional (4D) geometric quantities. From the viewpoint of such formulation there is no EDM-induced magnetization  $M$ ; in the 4D spacetime the EDM  $d^a$  is not proportional to  $S^a$ . It is argued that the induced  $M$  can come from the direct interaction between the applied electric field  $E^a$  and a MDM  $m^a$ .

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## I. INTRODUCTION

In [1], the paramagnetic insulating sample is subjected to an electric field  $\mathbf{E}$ , see the Introduction and Fig. 1 in [1]. It is supposed in [1], as usual in the elementary particle theories, that not only the magnetic dipole moment (MDM)  $\mathbf{m}$  is proportional to the spin  $\mathbf{S}$  ( $\mathbf{m} = m(\mathbf{S}/S)$ , the Uhlenbeck-Goudsmit hypothesis), but the electric dipole moment (EDM)  $\mathbf{d}$  as well (the usual 3-vectors are written in boldface). The external electric field  $\mathbf{E}$  orients permanent EDMs along the field; the interaction term is  $-\mathbf{d} \cdot \mathbf{E} = -d(\mathbf{S}/S) \cdot \mathbf{E}$ . Hence, the MDMs will be oriented as well. It will cause that the sample acquires a net magnetization  $\mathbf{M}$  that is measured by a SQUID magnetometer, as the electric field is reversed. The interaction between the electric field  $\mathbf{E}$  and a MDM  $\mathbf{m}$  is only indirect through the alignment of three-dimensional (3D) spins by the interaction  $-d(\mathbf{S}/S) \cdot \mathbf{E}$ . Thus, the EDM-induced magnetization  $\mathbf{M}$  is obtained

$$M = \chi k d_e E / \mu_a, \quad (1)$$

Eq. (1) in [1];  $M$  is determined by the electron EDM  $\mathbf{d}_e$  and the applied electric field  $\mathbf{E}$ . Hence, measuring the magnetization the authors also indirectly measured the electron EDM.

Recently, [2], the Uhlenbeck-Goudsmit hypothesis is generalized in a Lorentz covariant manner using 4D geometric quantities; the dipole moment tensor  $D^{ab}$  is proportional to the spin four-tensor  $S^{ab}$ ,  $D^{ab} = g_S S^{ab}$ , Eq. (9) in [2]. Using a general rule for the decomposition of a second rank antisymmetric tensor,  $D^{ab}$  is decomposed according to Eq. (2) in [2]. The dipole moment vectors  $d^a$  and  $m^a$  are then derived from  $D^{ab}$  and the velocity vector of the particle  $u^a$  ( $d^a$ ,  $m^a$ ,  $u^a$ ,  $S^a$ , etc. are usually called 4-vectors). Similarly,  $S^{ab}$  is decomposed according to Eq. (8) in [2]. The usual “space-space” intrinsic angular momentum, spin  $S^a$ , and a new one, the “time-space” intrinsic angular momentum, spin  $Z^a$ , are derived from  $S^{ab}$  and  $u^a$ , Eq. (8) in [2]. Then, Eq. (10) in [2] is obtained as

$$m^a = c g_S S^a, \quad d^a = g_S Z^a, \quad (2)$$

According to (2), the intrinsic MDM  $m^a$  of an elementary particle is determined by  $S^a$ , whereas the intrinsic EDM  $d^a$  is determined by the new spin vector  $Z^a$  and not, as usual, by the spin  $\mathbf{S}$ . Both spins,  $S^a$  and  $Z^a$ , are equally good physical quantities. The EDM  $d^a$ , which is obtained in this way, i.e., from the connection with the spin  $Z^a$ , Eq. (2), is an intrinsic property of elementary particles in the same way as it is the MDM  $m^a$ . In contrast with it, in the elementary particle theories, as mentioned in [2]: “.. an EDM is obtained by a dynamic calculation and it stems from an asymmetry in the charge distribution inside a fundamental particle, which is thought of as a charged cloud.” The EDM direction is connected with a net displacement of charge along the spin axis,  $\mathbf{d} = d\mathbf{S}/S$ . The reason for the assumption that  $\mathbf{d}$  has to be parallel to the spin  $\mathbf{S}$  comes from the general belief that  $\mathbf{S}$  is the only available 3-vector in the rest frame of the particle. However, as noticed in [2]: “... neither the direction of  $\mathbf{d}$  nor the direction of the spin  $\mathbf{S}$  have a well-defined meaning in the 4D spacetime. The only Lorentz-invariant condition on the directions of  $d^a$  and  $S^a$  in the 4D spacetime is  $d^a u_a = S^a u_a = 0$ . This condition does not say that  $\mathbf{d}$  has to be parallel to the spin  $\mathbf{S}$ .” Obviously, the same remark holds if  $\mathbf{d}$  is replaced by  $\mathbf{m}$  and  $d^a$  by  $m^a$ . More generally, from the viewpoint of the geometric approach from [2], the 3D quantities  $\mathbf{m}$ ,  $\mathbf{d}$ ,  $\mathbf{S}$ ,  $\mathbf{E}$ ,  $\mathbf{B}$ , etc. are not well-defined quantities in the 4D

spacetime and they have to be replaced by the 4D quantities,  $m^a$ ,  $d^a$ ,  $S^a$ ,  $E^a$ ,  $B^a$ , etc. The results from [2] are in a complete agreement with the symmetry of the 4D spacetime. They strongly indicate that the basic points of the interpretation of measurements of EDM in [1], i.e., both  $\mathbf{m}$  and  $\mathbf{d}$  are parallel to  $\mathbf{S}$ , are meaningless in the manifestly covariant formulation from [2].

This means that *in the Lorentz covariant formulation with 4D geometric quantities it is not possible to infer anything about the electron EDM  $d_e^a$  from measurements of magnetization of a paramagnetic insulating sample that is subjected to an electric field.* Instead of an indirect interaction between the applied electric field  $\mathbf{E}$  and a magnetic dipole moment  $\mathbf{m}$  through the alignment of 3D spins by the interaction  $-d(\mathbf{S}/S) \cdot \mathbf{E}$  we propose a direct, Lorentz covariant, interaction between the applied electric field  $E^a$  and a MDM  $m^a$ ; the term in  $L_{int}$  that is proportional to  $E_i m_k$ , Eq. (3) below. This new, Lorentz covariant, interaction that can be used for the interpretation of measurements of EDM in [1] will be exposed below using the results from [2].

## II. LORENTZ COVARIANT INTERACTION BETWEEN $F^{ab}$ AND $D^{ab}$

The interaction between the electromagnetic field  $F^{ab}$  and  $D^{ab}$  is given by the expression  $(1/2)F_{ab}D^{ba}$ , Eq. (3) in [2]. When the decomposition of  $F^{ab}$  (in terms of vector fields  $E^a$ ,  $B^a$  and the velocity vector  $v^a$  of the observers who measure fields), Eq. (1) in [2], and the above mentioned decomposition of  $D^{ab}$ , Eq. (2) in [2], are inserted into that expression then Eq. (3) in [2] is obtained. That equation is first reported in [3].

As can be seen from the discussion of Eqs. (1) and (2) in [2], when it is taken that the laboratory frame is the  $e_0$ -frame (the frame in which the observers who measure  $E^a$  and  $B^a$  are at rest with the standard basis  $\{e_\mu\}$  in it), then the temporal components of  $E^a$  and  $B^a$  will be zero,  $E^0 = B^0 = 0$ , and *only* three spatial components  $E^i$  and  $B^i$  will remain. Similarly, *only* in the particle's rest frame with the standard basis in it the dipole moments  $d^a$  and  $m^a$  will have  $d^0 = m^0 = 0$  and only three spatial components  $d^i$  and  $m^i$  will remain. Thus, it is not possible that, e.g., in the laboratory frame, *both*, the fields and the dipole moments have *only* three spatial components, i.e., as for the usual 3-vectors. Thus, for example, in all EDM experiments the interaction between the electromagnetic field and the dipole moments is described in terms of the 3-vectors as  $\mathbf{E} \cdot \mathbf{d}$  and  $\mathbf{B} \cdot \mathbf{m}$ .

Furthermore, it can be seen from the discussion of Eq. (25) in [2] that in the laboratory frame, as the  $e_0$ -frame, we can neglect the contributions to  $L_{int}$  from the terms with  $d^0$  and  $m^0$ ; they are  $u^2/c^2$  of the usual terms  $E_i d^i$  or  $B_i m^i$ . Then, what remains from Eq. (3) in [2] is

$$L_{int} = -((E_i d^i) + (B_i m^i)) - (1/c^2)\varepsilon^{0ijk}(E_i m_k - c^2 B_i d_k)u_j. \quad (3)$$

This is, to order  $0(u^2/c^2)$ , relativistically correct expression with 4D vectors for  $L_{int}$ . The last two terms that contain the direct interactions between  $E^a$  and  $m^a$  and between  $B^a$  and  $d^a$  are not taken into account in any of the EDM searches. With the usual 3-vectors, it would correspond to Eq. (26) in [2]. But, as stated at the end of Sec. 5 in [2]: “In the 4D geometric approach presented in this paper the expressions like (26), (28) and (29) are meaningless, because, as explained particularly in [12], there are not the usual time-dependent 3-vectors in the 4D spacetime.” Namely, [2]: “... what is essential for the number of components of a vector field is the number of variables on which that vector field depends, i.e., the dimension of its domain. Thus, strictly speaking, the time-dependent  $\mathbf{E}(\mathbf{r}, t)$  and  $\mathbf{B}(\mathbf{r}, t)$  cannot be the 3-vectors, since they are defined on the spacetime.” Hence, contrary to the opinion of majority of physicists, the usual formulation with 3-vectors  $\mathbf{E}$ ,  $\mathbf{B}$ ,  $\mathbf{S}$ , etc. **IS NOT** relativistically correct formulation.

It is seen from the above Eq. (3) for  $L_{int}$  that the interaction between the applied electric field and an EDM is contained in the term  $-E_i d^i$ . But, according to Eq. (2)  $d^a$  is determined by the “time-space” spin  $Z^a$ , and not by the 3-vector spin  $\mathbf{S}$ . Furthermore, the interaction between the applied electric field and a 4D magnetic moment  $m^a$ , which is determined by the 4D spin  $S^a$ , is contained in the term  $-(1/c^2)\varepsilon^{0ijk}E_i m_k u_j$ , which is  $u^a$ -dependent. Hence, according to this formulation, it is again visible that there is no EDM-induced magnetization (the interaction term  $-(1/c^2)\varepsilon^{0ijk}E_i m_k u_j$  does not contain  $d^a$ ), but only EDM-induced polarization (by means of  $-E_i d^i$  term). This consideration also shows that in the manifestly covariant formulation of the interaction between the electric and magnetic fields and the dipole moments *the magnetization induced by the applied electric field can be explained only by the term  $-(1/c^2)\varepsilon^{0ijk}E_i m_k u_j$ .*

The same consideration can be completely applied to the recent magnetization-based EDM search of the same authors that is presented in [7].

## III. SOME ADDITIONAL REMARKS

It is declared by S. K. Lamoreaux, [4]: “In the mean time, I’ll use the usual formulation of three vectors for the low velocities that we use in EDM experiments. As the sources are not moving relative to the boundaries of the experiment, this formulation is correct.”

However, it has to be emphasized that the usual 3-vectors, e.g.,  $\mathbf{E}$  and  $\mathbf{B}$  **ARE NOT** the low-velocity approximation of the 4D vectors  $E^a$  and  $B^a$  and the usual transformations (UT) of  $\mathbf{E}$  and  $\mathbf{B}$ , Eq. (6) in [2], or Eqs. (11.148) and (11.149) from Jackson’s book (Ref. [10] in [2]), **ARE NOT** the low-velocity approximation of the Lorentz transformations (LT) of the 4D vectors  $E^a$  and  $B^a$ , Eq. (7) in [2]. Namely, according to the LT, e.g., the components of the electric field 4D vector will be always transformed again into the components of the electric field 4D vector; there is no mixing with the components of the magnetic field 4D vector. It is just the opposite in the UT. For more detail about the fundamental difference between the UT and the LT of the electric and magnetic fields see, e.g., [5] or [6]. In [6], it is shown that Minkowski first discovered the correct LT of the 4D electric and magnetic fields, see also Ref. [12] in [2].

In addition, it is worthwhile to mention that in the approach from [2], see the discussion in Sec. 5, neither the  $T$  inversion nor the  $P$  inversion are good symmetries in the 4D spacetime, because they are synchronization dependent.

#### IV. CONCLUSIONS

The consideration presented here shows that Eq. (1) from [1] (Eq. (1) here), according to which the magnetization  $M$  is determined by the electron EDM  $\mathbf{d}_e$ , is not properly justified from the point of view of the manifestly covariant formulation of the interaction between the electromagnetic field and the dipole moments. It is argued that the interaction term that is responsible for the induced magnetization by the applied electric field in experiments in [1] is given by the third term in  $L_{int}$ , Eq. (3), which does not contain the electron EDM  $d^a$ .

This means that, *in order to get some useful informations about the electron EDM in the experiments from [1] (and [7]), one would need to measure a voltage induced by polarization due to the term  $-(E_i d^i)$  in  $L_{int}$ , Eq. (3), and not the magnetization, because  $M$  cannot give any information about EDMs.*

Further examination of these results and their comparison with experiments from, e.g., [1] (and [7]), requires much more work together with the experimentalists who search for the electron EDM. Our aim was to explain that the underlying physics (the 3D  $\mathbf{m}$  and  $\mathbf{d}$  are both parallel to  $\mathbf{S}$ ) in experiments in [1] (and [7]), and in other experimental searches for the electron EDM, is not well-founded in the 4D spacetime.

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